A 11

classmate

Date

Page

# LINEARLY DEP. & INDEP. VECTORS.

Let V(F) be a V.S. over the field F. The vectors $x_1, x_2, \dots, x_n$ are said to be linearly Dep. if $\exists$ $\alpha_1, \alpha_2, \dots \alpha_n \in F$ (not all zero). s.t.

$$\alpha_1 x_1 + \alpha_2 x_2 + \dots + \alpha_n x_n = 0$$

if all $\alpha_i$'s are zero, then it is L.I.

**Ques.** Let $\mathbb{R}^2(\mathbb{R})$ be a V.S. over the field $\mathbb{R}$. The vectors $(1,0)$, $(0,1)$ and $(1,1)$ are L.I. or L.D.?

$$\alpha(1,0) + \alpha_2(0,1) + \alpha_3(1,1) = (0,0)$$

$$(\alpha_1 + \alpha_3, \alpha_3 + \alpha_2) = (0,0)$$

$$\alpha_1 = -\alpha_3 \;;\; \alpha_3 = -\alpha_2 \Rightarrow \alpha_1 = \alpha_2 = -\alpha_3$$

$\therefore$ L.D.

2nd Method:
$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

**#Q.** Let $V(\mathbb{R}) = P_3(\mathbb{R})$ be a V.S. The vectors $1+x$, $x$, $x^2$ are L.I. or L.D.

$$\alpha_1(1+x) + \alpha_2 x + \alpha_3 x^2 = 0$$

$$\alpha_1 + (\alpha_1 + \alpha_2)x + \alpha_3 x^2 = 0 + 0x + 0x^2$$

$$\alpha_1 = 0$$

$$\alpha_1 + \alpha_2 = 0 \Rightarrow \alpha_1 = -\alpha_2 = 0, \quad \alpha_3 = 0$$

$\therefore$ ~~L.D~~ L.I.

**Th:** Let V(F) be a V.S. over the field F. The vectors $x_1, x_2, \dots, x_n$ are said to be L.D. iff one of the vector can be written as a l.c. of others, i.e. $x_i = \sum_{j \neq i} \beta_j x_j$